
Countermeasures against Differential Power Analysis for Hyperelliptic Curve Cryptosystems

Roberto Avanzi

`mocenigo@exp-math.uni-essen.de`

IEM – University of Duisburg–Essen

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<http://www.arihcc.com>

In full screen mode, click on titles to go to corresponding slide.

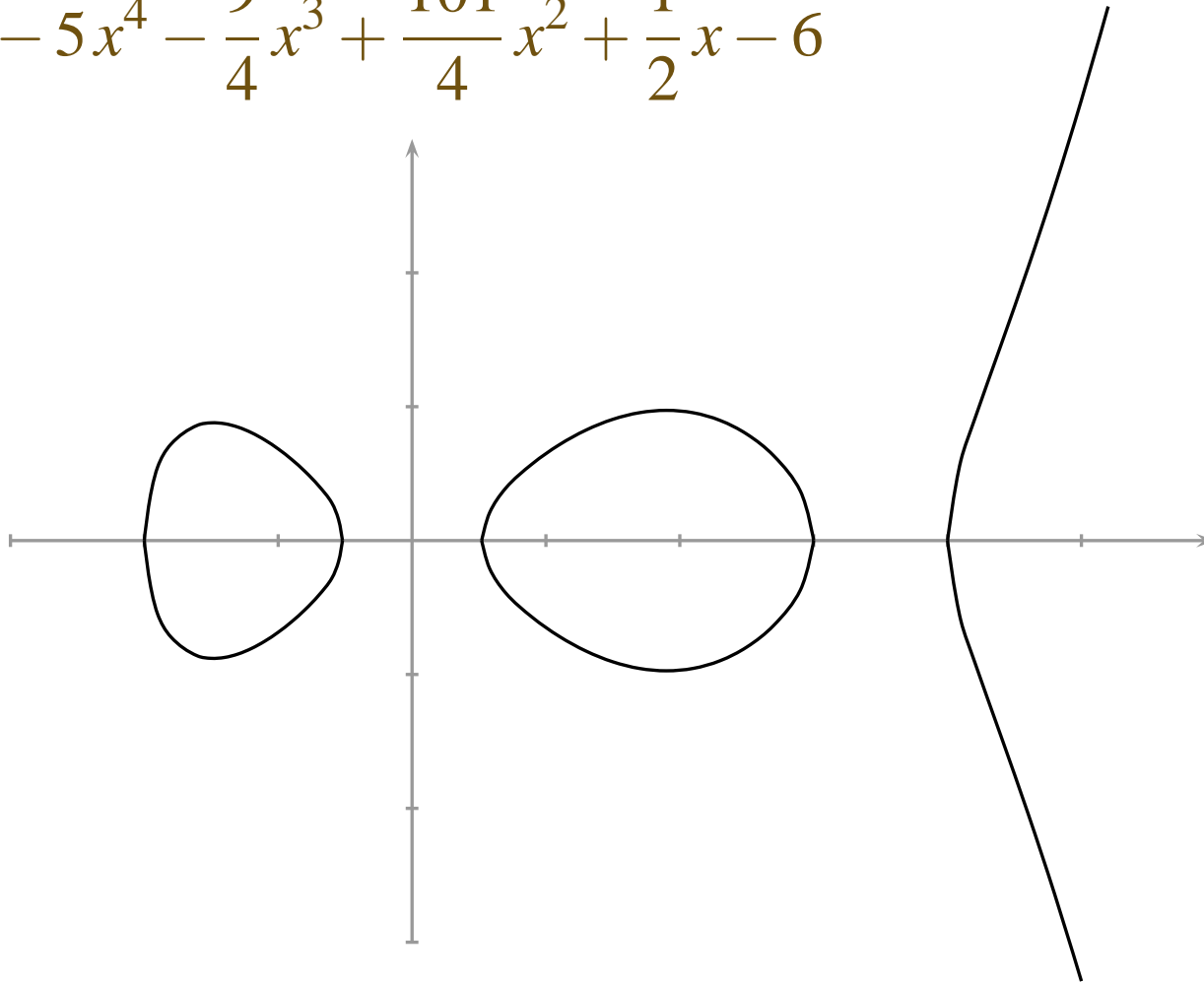
- Why HECC
- Group of Divisors
- Divisor Classes and Mumford Representation
- Dangers for HECC
 - Simple Side Channel Analysis
 - Differential Side Channel Analysis and countermeasures
 - Goubin-type attacks and countermeasures
- Typos in Paper
- Conclusions

- Not wise to put all eggs in one basket.
- hecc close to ecc in **performance**: $\pm 10\%$.
See Pelzl, et al., before lunch, for $g = 3$ over binary fields, and (in progress) A. for $g = 2$ over prime fields.
- Smaller fields might allow use of **cheaper** hardware (but: put more software on card.)
- For the moment, less patents for hecc than on ecc.

Just a reminder...

Here's how a hyperelliptic curve C of genus 2 looks like!

$$y^2 = x^5 - 5x^4 - \frac{9}{4}x^3 + \frac{101}{4}x^2 + \frac{1}{2}x - 6$$



Group of Divisors

Curve $C : y^2 + h(x)y = f(x)$

f monic, $\deg f = 2g + 1$, $\deg h \leq g$. $g = \text{genus}$.

Points on a hyperelliptic curve in general do **not** form a group!

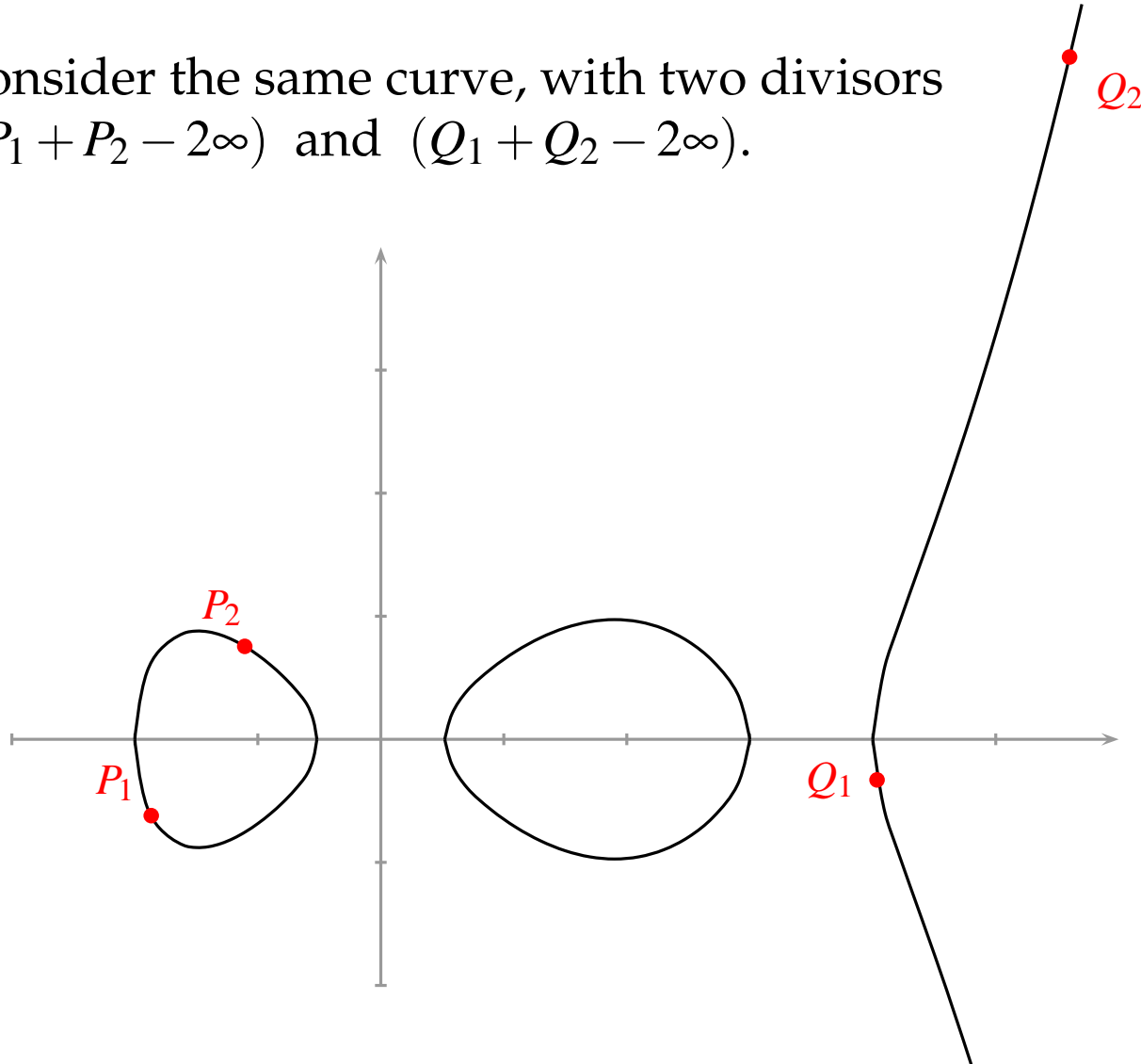
Use **divisors**, i. e. “sets of points” with multiplicities:

$$\sum_{i=1}^k m_i P_i - \left(\sum_{i=1}^k m_i \right) \infty \quad : \quad m_i > 0, \quad P_i \in C \setminus \{\infty\}$$

We show how this works “geometrically”.

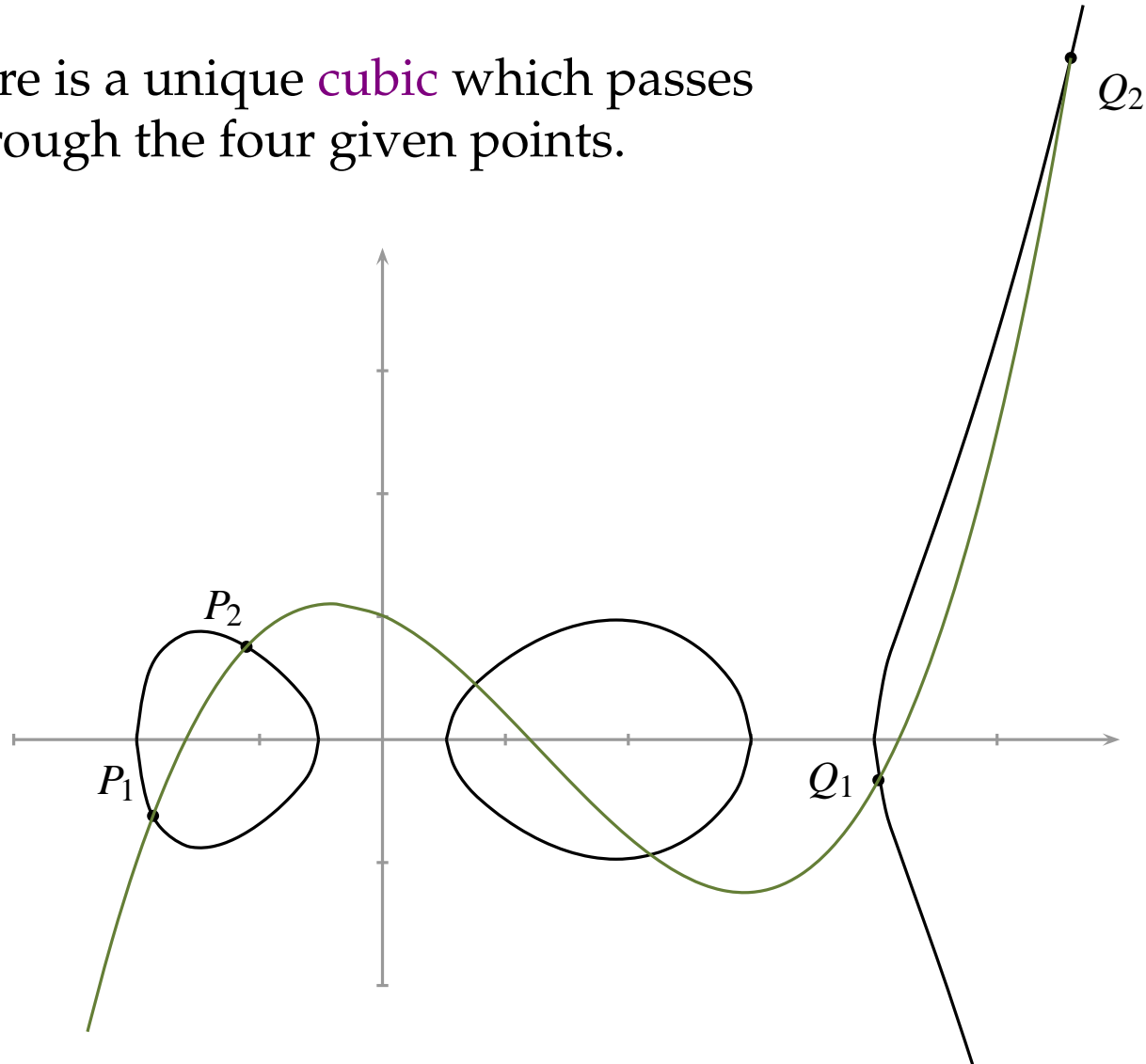
How to do HECC

Consider the same curve, with two divisors
 $(P_1 + P_2 - 2\infty)$ and $(Q_1 + Q_2 - 2\infty)$.



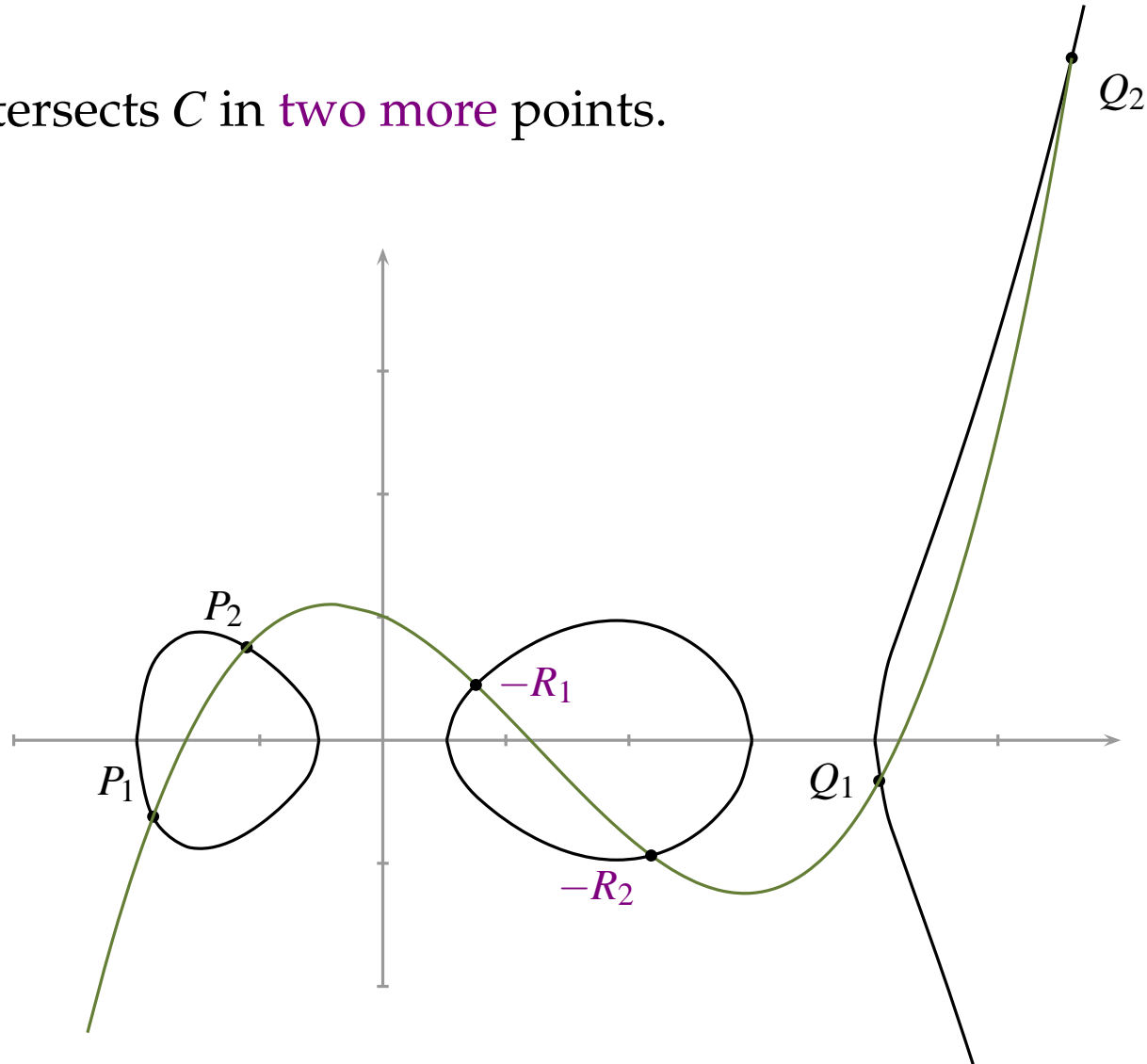
How to do HECC

There is a unique **cubic** which passes through the four given points.



How to do HECC

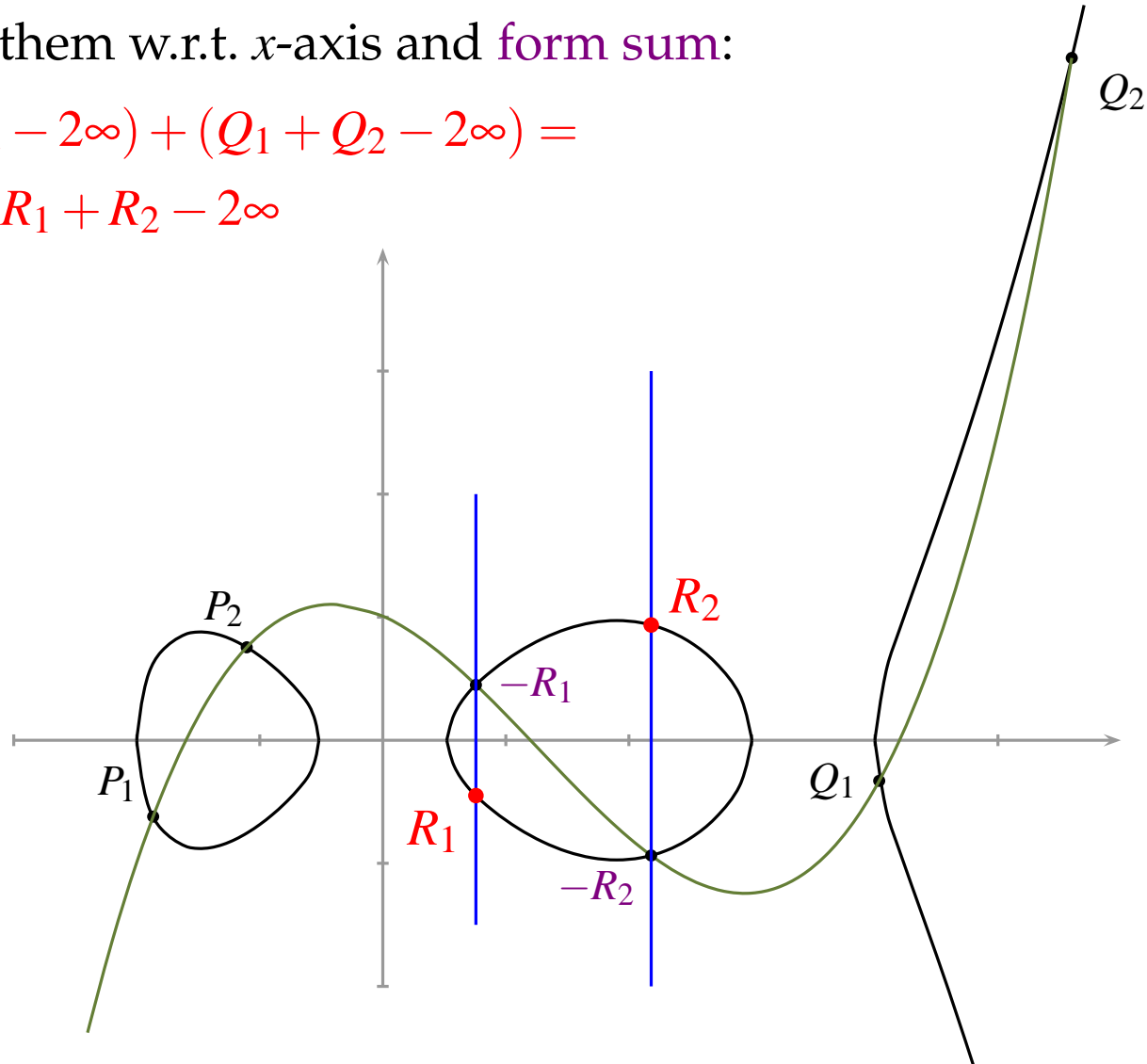
It intersects C in **two more** points.



How to do HECC

Mirror them w.r.t. x -axis and form sum:

$$(P_1 + P_2 - 2\infty) + (Q_1 + Q_2 - 2\infty) = \\ = R_1 + R_2 - 2\infty$$



Divisor Classes and Mumford Representation

This defines a group, the **Jacobian** of C , $\text{Jac}(C)$.

If $\mathcal{K} = \mathbb{F}_q$, then $\#\text{Jac}(C) \approx q^g$.

But working with “point sets” and intersecting curves is **very inefficient**.

Better:

- Mumford representation and
- Cantor’s algorithm \Rightarrow explicit formulæ.

Divisor Classes and Mumford Representation

Curve $C : y^2 + h(x)y = f(x)$

Let $D = \sum m_P P - (\sum m_P) \infty$ have $\deg \sum m_P \leq g$.
– more precisely: degree of associated effective divisor –

D represented by **unique** pair of polynomials
 $U(t), V(t) \in \mathcal{K}[t]$ with: $g \geq \deg_t U > \deg_t V$, U monic.

$$\begin{cases} U(t) = \prod (t - x_P)^{m_P} \\ V(x_P) = y_P \text{ for all } P \\ U(t) \text{ divides } V(t)^2 + V(t)h(t) - f(t) \end{cases}$$

Coordinates of D : **the coefficients of U and V** .

Two categories of attacks:

- *mathematical* (on structure) and
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Here we consider *Side Channel Analysis*.

- Simple.
- Differential.
- Goubin type.

I will not describe them for the umpteenth time here...

Not interested in fault analysis in this paper.

Simple Side Channel Analysis

Solution: make sequence of elementary ops regular.

- Make sequence of group ops **homogeneous** (e.g. Coron's double-and-add-always).
- Make the group ops **indistinguishable** (e.g. Hess or Jacobi form for ecc, Brier-Joye, insertion of dummy ops: latter easy with Lange's genus 2 formulae); or split the group ops into blocks which can be made **regular** (Ciet-Joye).

From now assume hecc immunised against SPA.

Differential Side Channel Analysis

Applies to computations $n \cdot D$ in the group G , n fixed.

Exploits knowledge of internal representation of operands.

⇒ **internal data must be unpredictably scrambled:**

Some techniques for previous cryptosystems:

Faster

- Joye-Tymen (ecc): Compute in isomorphic curve.
- Coron's 2nd and 3rd (ecc): Randomise D .

Slower

- Coron's 1st: Randomise scalar n .
- Joye-Tymen: Use isomorphic binary field.

Which countermeasures for hecc?

First Countermeasure: *Curve Randomisation*

hecc analogue of Joye-Tymen's ecc curve randomisation.

$\phi : \mathcal{C} \rightarrow \tilde{\mathcal{C}}$ = a \mathcal{K} -isomorphism of hyperelliptic curves.

\Rightarrow \mathcal{K} -isomorphism $\phi : \text{Jac}(\mathcal{C}) \rightarrow \text{Jac}(\tilde{\mathcal{C}})$.

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Transfer all points of D over \mathcal{C} to $\tilde{\mathcal{C}}$ "simultaneously" by manipulating coordinates of divisor.

First Countermeasure: Curve Randomisation

$$\begin{array}{ccc} D \in \text{Jac}(\mathcal{C})(\mathcal{K}) & \xrightarrow{\text{multiplication by } n} & \text{Jac}(\mathcal{C})(\mathcal{K}) \ni n \cdot D \\ \phi \downarrow & & \uparrow \phi^{-1} \\ \phi(D) \in \text{Jac}(\tilde{\mathcal{C}})(\mathcal{K}) & \xrightarrow{\text{multiplication by } n} & \text{Jac}(\tilde{\mathcal{C}})(\mathcal{K}) \ni n \cdot \phi(D) \end{array}$$

Details in the paper. Two types of isomorphisms:

Using only multiplications: *All coefficients of \mathcal{C} and of D are multiplied by different powers of a randomly chosen $s \in \mathcal{K}$.*

Total # field muls LESS than in one group op!

Using also additions: everything can become slower.
(work with more general curves).

Second Countermeasure: *Divisor Randomisation*

On embedded hardware, field inversion is very slow.

This prompted the introduction of *projective coordinates*.
They do not require inversions.

A group element has many different representations.

For ecc: two triples (X, Y, Z) and (sX, sY, sZ) represent the same point if $s \in \mathcal{K}^\times$.

Coron uses them to randomise the base point:
replaces (X, Y, Z) with (sX, sY, sZ) for a random $s \in \mathcal{K}^\times$.

Second Countermeasure: *Divisor Randomisation*

For genus 2 hecc: Projective and New coordinates (Lange).

Projective: a divisor $D \equiv [U(t), V(t)]$ is represented as a quintuple $[U_1, U_0, V_1, V_0, Z] \in \mathcal{K}^5$ where

$$U(t) = t^2 + \frac{U_1}{Z}t + \frac{U_0}{Z} \quad \text{and} \quad V(t) = \frac{V_1}{Z}t + \frac{V_0}{Z} .$$

The randomisation consists in picking a random $s \in \mathcal{K}^\times$ and by performing the following replacement

$$[U_1, U_0, V_1, V_0, Z] \mapsto [sU_1, sU_0, sV_1, sV_0, sZ] .$$

For New coordinates the method is entirely similar.

Goubin type attacks: Context

Remark: randomisation of zero by multiplication by a random value, or by random isomorphism, is... zero!

Definition: context of Goubin-type attacks:

Let H be a *small subset* of the group G s.t.:

- The *elements of H* possess properties which makes their processing *detectable by side-channel analysis* – for example, zeros in the internal representation – and
- are *invariant under a given randomisation procedure R* .

H := set of special points/divisors.

Goubin type attacks: *Description*

Suppose most significant digits $n_r, n_{r-1}, \dots, n_{j+1}$ of n known;
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Then, statistical correlation of side-channel traces may reveal if the guess was correct even if R is used.

Goubin type attacks: *The bad news*

Such sets H exist. Examples:

- $H =$ Points with a zero coordinate of an elliptic curve.
- $H =$ Divisors on a hyperelliptic curve with a zero coordinate (e.g. of $\deg < g$).

Preserved by above randomisations.

Probability random point/divisor $\in H$ is $O(q^{-1})$, $q = \#\mathcal{K}$, so set is small.

On ecc it is easy to avoid such points (remember Nigel Smart's talk). **But for hecc?**

Goubin type attacks: *The good news*

Scalar randomization: ok (but: slow).

Message blinding: hecc analogue of Coron's 2nd method.

R = secret divisor, with $S = n \cdot R$ known.

Compute $n \cdot (D + R) - S$ in place of $n \cdot D$.

If R belongs to the group generated by D (normal case),
equivalent to *isogeny* of random degree:

if $R = m \cdot D$ then $D + R = (m + 1) \cdot D$.

An isogeny is *not* an isomorphism with probability
 $1 - O(q^{-1}) \Rightarrow$ images of "special" divisors are not special.

Page 378, line 7.

ERRATA: ... $\deg(t(D + R)) = g$ also with probability $O(q^{-1})$...

CORRIGE: ... $\deg(t(D + R)) < g$ also with probability $O(q^{-1})$...

Conclusions

- Two methods to prevent basic DPA for hecc.
 - Curve randomisation (generic).
 - Divisor randomisation (specific).
 - Cheaper than a single group operation!
- Serious Goubin-type attacks on hecc discovered.
 - Suitable divisor randomisation to thwart them.
 - Costs as few group operations.

Ditto for trace-zero varieties (Frey, Naumann, Lange, Lange-A.).

Now one can really start deploying hecc on embedded devices!

Any questions?

